Spherical chirolenses

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The properties of a spherical lens made from chiral media are examined, using a rigorous electromagnetic approach based on dyadic Green's functions. It is shown that separate focal points exist for each of the two circularly polarized eigenmodes present in the medium. Furthermore it is demonstrated that the lens can focus one of the modes while defocusing the other.

Within the past few years it has become increasingly apparent that a variety of new optical, millimeter-wave, and microwave devices may be devised by employing an optically active medium, often called a chiral medium, instead of an ordinary dielectric. These devices have been suggested by the recent investigations of chirowaveguides[1–4] and those of electromagnetic sources embedded in unbounded chiral media.[5] The novel possibilities offered by these studies have prompted us to examine the spherical *chirolens*, a homogeneous spherical lens made from chiral materials. Our primary purposes are twofold: to show, from electromagnetic theory, that chirolenses possess two distinct focal points and to demonstrate another application of the dyadic Green's functions that we had previously derived.[6]

In general, the electromagnetic properties of a homogeneous isotropic chiral medium may be described by the constitutive relations[7] $\mathbf{D} = \epsilon_c \mathbf{E} + i\xi_c \mathbf{B}$ and $\mathbf{H} = i\xi_c \mathbf{E} + \mathbf{B}/\mu_c$. The quantities ϵ_c, μ_c, and ξ_c are, respectively, the permittivity, the permeability, and the chirality admittance of the medium. The additional chirality admittance parameter is introduced to account for the new characteristics resulting from the medium's handed constituents. From Maxwell's equations and these constitutive relations, it can be seen that chiral media possess a polarization birefringence with circularly polarized eigenmodes. These eigenmodes propagate with wave numbers $k_\pm = \pm\omega\mu_c\xi_c + (\omega^2\mu_c{}^2\xi_c{}^2 + \omega^2\mu_c\epsilon_c)^{1/2}$, where k_+ corresponds to the right-circularly polarized (RCP) mode and k_- corresponds to the left-circularly polarized (LCP) mode.

Here we examine the properties of the chirolens by placing an electric-dipole source at its exterior. However, if instead the dipole were located inside the lens, the results and analysis would be similar to what follows. As shown in Fig. 1, the geometry under consideration consists of a homogeneous sphere of radius a made from an optically active material. The sphere is characterized by the electrical parameters ϵ_c, μ_c, and ξ_c and is embedded in a simple dielectric with permittivity ϵ and permeability μ. Thus the interior of the sphere supports both k_+ and k_-, while its exterior supports only $k = \omega(\epsilon\mu)^{1/2}$. The source is directed along the x axis and is located at a distance $b > a$ from the origin. Its excitation is

$$\mathbf{J}(\mathbf{r}') = \mathbf{e}_\theta \frac{I_o \delta(r' - b)\delta(\theta')\delta(\phi')}{r'^2 \sin\theta'}, \tag{1}$$

with the spherical coordinates (r, θ, ϕ) and the related unit vectors also depicted in Fig. 1. It should be noted that the unprimed coordinates denote the observation point, whereas the primed ones represent the location of the source (in the remainder of this Letter any primed variable refers to the location of the source).

Using the dyadic Green's function for a homogeneous chiral sphere,[6] we express the electric field outside the sphere as

$$\mathbf{E}(\mathbf{r}) = i\omega\mu \int \underline{\underline{\Gamma}}_{\mathrm{tot}}{}^{(11)}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') \mathrm{d}V', \tag{2}$$

where, for all observation points with $r > b$, $\underline{\underline{\Gamma}}_{\mathrm{tot}}{}^{(11)}(\mathbf{r}, \mathbf{r}')$ is given by[6]

$$\begin{aligned}\underline{\underline{\Gamma}}_{\mathrm{tot}}{}^{(11)}(\mathbf{r}, \mathbf{r}') = {}& \frac{i}{4\pi k} \sum_{n=1}^{\infty} \sum_{m=0}^{n} (2 - \delta_{m0}) \frac{(2n+1)}{n(n+1)} \frac{(n-m)!}{(n+m)!} \\ & \times \{[e_n{}^v \mathbf{V}_{\substack{e\\o}mn}{}^{(1)\prime}(k) + e_n{}^w \mathbf{W}_{\substack{e\\o}mn}{}^{(1)\prime}(k) \\ & + k^2 \mathbf{V}_{\substack{e\\o}mn}{}'(k)] \mathbf{V}_{\substack{e\\o}mn}{}^{(1)}(k) + [f_n{}^v \mathbf{V}_{\substack{e\\o}mn}{}^{(1)\prime}(k) \\ & + f_n{}^w \mathbf{W}_{\substack{e\\o}mn}{}^{(1)\prime}(k) + k^2 \mathbf{W}_{\substack{e\\o}mn}{}'(k)] \mathbf{W}_{\substack{e\\o}mn}{}^{(1)}(k)\}.\end{aligned} \tag{3}$$

The spherical vector wave functions $\mathbf{V}_{\substack{e\\o}mn}(k)$ and $\mathbf{W}_{\substack{e\\o}mn}(k)$ are simply related[6] to the usual $\mathbf{M}_{\substack{e\\o}mn}(k)$ and $\mathbf{N}_{\substack{e\\o}mn}(k)$. Written explicitly,

$$\begin{aligned}\mathbf{V}_{\substack{e\\o}mn}(k) = \frac{1}{\sqrt{2}} \Bigg(& \mp \frac{m}{\sin\theta} P_n{}^m(\cos\theta) {}^{\sin}_{\cos}(m\phi) \\ & \times \left\{ j_n(kr)\mathbf{e}_\theta + \frac{1}{kr}\frac{\partial}{\partial r}[r j_n(kr)]\mathbf{e}_\phi \right\} + \frac{\partial P_n{}^m(\cos\theta)}{\partial\theta} \\ & \times {}^{\cos}_{\sin}(m\phi) \left\{ \frac{1}{kr}\frac{\partial}{\partial r}[r j_n(kr)]\mathbf{e}_\theta - j_n(kr)\mathbf{e}_\phi \right\} \\ & + n(n+1) P_n{}^m(\cos\theta) {}^{\cos}_{\sin}(m\phi) \frac{j_n(kr)}{kr} \mathbf{e}_r \Bigg),\end{aligned} \tag{4}$$

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and $\mathbf{W}_{\substack{e\\o}mn}(k)$ is obtained from $\mathbf{V}_{\substack{e\\o}mn}(k)$ by exchanging $1/kr$ with $-1/kr$. The superscript (1) in Eq. (3) indicates that the spherical Bessel functions $j_n(kr)$ are replaced by the spherical Hankel functions $h_n^{(1)}(kr)$. The coefficients e_n^v, e_n^w, f_n^v and f_n^w are

$$\begin{Bmatrix} e_n^{\,v} \\ f_n^{\,w} \end{Bmatrix} = k^2\{j_+\partial j_-[(l \pm 1)^2\partial jh + (l \mp 1)^2 j\partial h] + \partial j_+ j_-[(l \pm 1)^2\partial jh + (l \mp 1)^2 j\partial h] - 4l(j_+j_-\partial j\partial h + \partial j_+\partial j_- jh)\}/2D, \tag{5a}$$

$$e_n^{\,w} = f_n^{\,v} = -k^2(l^2 - 1)(j_+\partial j_- + \partial j_+ j_-)(j\partial h - \partial jh)/2D, \tag{5b}$$

where $D = 2l(h^2\partial j_+\partial j_- + \partial h^2 j_+ j_-) - h\partial h(l^2 + 1)(j_+\partial j_- + j_-\partial j_+)$, and the following notation has been used:

$$j = j_n(ka), \qquad \partial j = \frac{1}{ka}\frac{\partial}{\partial r}[rj_n(kr)]|_a,$$

$$j_\pm = j_n(k_\pm a), \qquad \partial j_\pm = \frac{1}{k_\pm a}\frac{\partial}{\partial r}[rj_n(k_\pm r)]|_a,$$

$$h = h_n^{(1)}(ka), \qquad \partial h = \frac{1}{ka}\frac{\partial}{\partial r}[rh_n^{(1)}(kr)]|_a.$$

In Eqs. (5a) and (5b), $l = (\epsilon/\mu)^{1/2}/[\xi_c^2 + (\epsilon_c/\mu_c)]^{1/2}$ is the ratio of the intrinsic impedance inside the sphere to that outside. Noting that[8]

$$\left.\frac{P_n^{\,m}(\cos\theta')}{\sin\theta'}\right|_{\theta'=0} = \left.\frac{\partial P_n^{\,m}(\cos\theta')}{\partial\theta'}\right|_{\theta'=0} = \begin{cases} 0 & \text{if } m \neq 1 \\ -\dfrac{n(n+1)}{2} & \text{if } m = 1 \end{cases}$$

and performing the integration in Eq. (3), we obtain the complete expression for the electric field in the region $r > b$,

$$\begin{aligned} \mathbf{E}(r) = -\frac{\omega\mu I_0}{4\sqrt{2}\pi k}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}\Bigg(&\mathbf{V}_{e1n}^{(1)}(k)\left\{(-e_n^{\,v} + f_n^{\,v}) \times \frac{1}{kb}\frac{\partial}{\partial r}[rh_n^{(1)}(kr)]|_b - \frac{k}{b}\frac{\partial}{\partial r}[rj_n(kr)]|_b\right\} \\ &+ \mathbf{V}_{o1n}^{(1)}(k)[-(e_n^{\,v} + f_n^{\,v})h_n^{(1)}(kb) - k^2 j_n(kb)] \\ &+ \mathbf{W}_{e1n}^{(1)}(k)\left\{(-e_n^{\,w} + f_n^{\,w})\frac{1}{kb}\frac{\partial}{\partial r}[rh_n^{(1)}(kr)]|_b + \frac{k}{b}\frac{\partial}{\partial r}[rj_n(kr)]|_b\right\} + \mathbf{W}_{o1n}^{(1)}(k) \\ &\times[-(e_n^{\,w} + f_n^{\,w})h_n^{(1)}(kb) - k^2 j_n(kb)]\Bigg). \end{aligned} \tag{6}$$

The focal points can now be found by simplifying Eq. (6) with the help of the following asymptotic expansions, which are valid for $\rho \gg n \gg 1$:

$$h_{(n)}^{(1)}(\rho) \approx \frac{1}{\rho}\exp\{i[\gamma_n(\rho) - \pi(n+1)/2]\},$$

$$j_n(\rho) \approx \frac{1}{2\rho}(\exp\{i[\gamma_n(\rho) - \pi(n+1)/2]\} + \text{c.c.}), \tag{7}$$

where

$$\gamma_n(\rho) = \rho + \frac{\left(n + \frac{1}{2}\right)^2}{2\rho} + \frac{\left(n + \frac{1}{2}\right)^4}{24\rho^3} + \ldots. \tag{8}$$

These expansions have previously been used by Wu *et al.*[9] to examine nonchiral spherical lenses. Following an approach similar to that of Ref. 9, we now obtain the focal points of a chiral sphere. We limit our attention to the $\mathbf{e}_x$–$\mathbf{e}_z$ plane, i.e., $\phi = 0$, and to geometries and frequencies[10] for which relations (7) can be used to replace all the Bessel and Hankel functions in Eq. (6). Furthermore, we make the approximation that there are no reflections at the interface between the lens and the surrounding dielectric and thus set $l \approx 1$. Under these conditions the electric field in the far zone may be expressed in the form

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_+(\mathbf{r}) + \mathbf{E}_-(\mathbf{r}), \tag{9}$$

with the RCP and LCP components given by

$$\begin{aligned} \mathbf{E}_\pm(\mathbf{r}) = \frac{\omega\mu I_o}{16bkr}(\mathbf{e}_\theta \pm i\mathbf{e}_\phi)\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}\Bigg(&\exp\{i[\gamma_n(kr) - \gamma_n(kb)]\}\left[\frac{P_n^{\,1}(\cos\theta)}{\sin\theta} + \frac{\partial P_n^{\,m}(\cos\theta')}{\partial\theta'}\right] + (-1)^n \\ &\times \exp\{i[\gamma_n(kr) - 2\gamma_n(ka) + \gamma_n(kb) + 2\gamma_n(k_\pm a)]\} \\ &\left[-\frac{P_n^{\,1}(\cos\theta)}{\sin\theta} + \frac{\partial P_n^{\,m}(\cos\theta')}{\partial\theta'}\right]\Bigg). \end{aligned} \tag{10}$$

It should be noted that only amplitude terms that decay as $1/r$ have been retained in this expression.

Ideally, in the geometrical-optics limit, when the dipole source is at the focal point, the field on the opposite side of the lens should approach that of a plane wave. Therefore we restrict ourselves to $\theta = \pi$, for which Eq. (10) simplifies[8] to

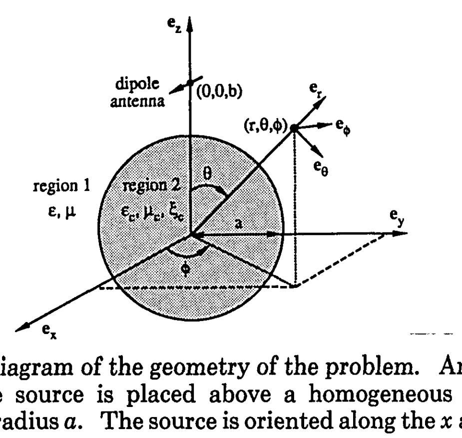

Fig. 1. Diagram of the geometry of the problem. An electric-dipole source is placed above a homogeneous chiral sphere of radius a. The source is oriented along the x axis at $(0, 0, b)$.

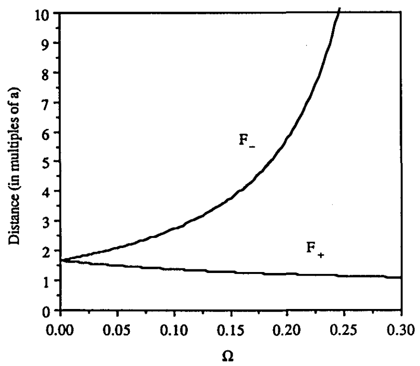

Fig. 2. Graph of the location of the two focal points as a function of chirality admittance. Here $\Omega = \xi_c(\mu_c/\epsilon_c)^{1/2}$, $\epsilon = \epsilon_c$, and $\mu/\mu_c = 0.7$.

$$E_{\pm}(\mathbf{r}) = -\frac{\omega\mu I_o}{8bkr}(\mathbf{e}_\theta \pm i\mathbf{e}_\phi) \times \exp\{i[k(r - 2a + b) + 2k_{\pm}a]\}S_{\pm}, \quad (11)$$

with

$$S_{\pm} \approx \sum_{q=3/2}^{\infty} q \exp\left\{\frac{iq^2}{2}\left(\frac{1}{kr} - \frac{2}{ka} + \frac{1}{kb} + \frac{2}{k_{\pm}a}\right) + \frac{iq^4}{24}\left[\frac{1}{(kr)^3} - \frac{2}{(ka)^3} + \frac{1}{(kb)^3} + \frac{2}{(k_{\pm}a)^3}\right]\right\}, \quad (12)$$

where $q = n + 1/2$. We need only retain terms with order q^2 in the exponential, since all higher-order terms are negligible. Furthermore, for most points of observation with $r \gg a$ and $r \gg b$, the r dependence in $S_{\pm}$ may be ignored. The exception occurs as b approaches one of the two focal points, denoted by F_+ and F_-. There we anticipate the r dependence to prevail as the field begins to resemble a paraxial plane wave. Hence the focal points must occur when the exponent is dominated by $1/kr$, that is, when $-2/ka + 1/kF_{\pm} + 2/k_{\pm}a = 0$ or

$$F_{\pm} = \frac{a}{2}\frac{k_{\pm}}{k_{\pm} - k}. \quad (13)$$

At these points we may approximate the summation in relation (12) by the integral

$$S_{\pm} \approx \int_0^{\infty} \exp(iq^2/2kr)\, q\, \mathrm{d}q = ikr\{1 - \exp[i\alpha(kr)]\}, \quad (14)$$

where $\alpha(kr)$ is introduced as an undeterminable phase term. Substituting relation (14) into Eq. (11), we obtain the form of the electric field on the opposite side of the lens,

$$\mathbf{E}_{\pm}(\mathbf{r}) \propto -\frac{i\omega\mu I_o}{8b} \times (\mathbf{e}_\theta \pm i\mathbf{e}_\phi)\exp\{i[k(r - 2a + b) + 2k_{\pm}a]\}. \quad (15)$$

Thus, by placing the source at either F_+ or F_-, we have been able to achieve (roughly) a plane wave for one of the chiral eigenmodes.

The location of the two points is plotted[11] as a function of ξ_c for $k_+ \geq k_- \geq k$ in Fig. 2. It is seen that the separation between F_+ and F_- increases with chirality. Furthermore, when $k_- = k$, the LCP focal point moves to infinity, indicating no focusing of that mode. As the chirality is increased even further, we reach the region $k_+ \geq k \geq k_-$ (not shown), where the LCP wave is defocused but the RCP wave remains focused. Hence the chirolens can serve simultaneously as a concave and a convex lens.

In this study we have shown that spherical chirolenses possess two distinct focal points for two eigenmodes by employing a wave-oriented approach based on dyadic Green's functions. It is worth noting that these focal points could have been obtained directly from geometrical optics. However, our analysis, unlike the ray-optics approach, can also be used to explain the behavior of electromagnetic fields when the dipole source is not situated at one of the focal points. The result can be generalized to an arbitrarily shaped lens made from chiral media and also to other frequency ranges such as infrared, microwave, and millimeter-wave regimes. These chirolenses could be used in a variety of new applications, such as couplers for waveguides (especially chirowaveguides), polarization filters, and new antennas for remote sensing. A possible arrangement for this last application could consist of a single chirolens with two antennas, a transmitter radiating a circularly polarized wave with one handedness at one focal point and a receiver detecting the return signal of a circularly polarized wave with the opposite handedness at the other. Such applications are currently under study.

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9. T. T. Wu, R. W. P. King, and H. Shen, J. Appl. Phys. **62**, 4036 (1987).
10. In order to apply the expansion to all the functions, we require that (ka, $k_{\pm}a$, kb, and kr) $\gg n \gg 1$, which can certainly be achieved in the geometric-optics regime, where we let $\omega \to \infty$.
11. The graphs of F_+ and F_-, and related comments, should be interchanged when ξ_c is negative.